Adams Operations and Representations of the Symmetric Group

ABSTRACT.

The development and study of topological "K-theory" in the 1960's led to some astonishing results. One the most famous conclusions gives an answer to the question whether it is possible to define a "nice" multiplication of vectors in $\mathbb{R}^n$ that allows for dividing by non-zero vectors:

Theorem If $\mathbb{R}^n$ is a division algebra or S^{n-1} has $n-1$ linearly independent (continuous) tangent vector fields then $n = 1, 2, 4, 8$.

It is not hard to see that if $\mathbb{R}^n$ satisfies the hypothesis of the theorem, there must be certain maps of "Hopf invariant one", as will be discussed in detail in the preceding talk by Markus Hausmann.

The most important tools for the solution of this "Hopf invariant one" problem are the so-called Adams operations, named after *John Frank Adams* and defined on topological K-theory, and the relations they fulfil. They are usually defined (very easily) via polynomials in the exterior powers of vector bundles but then a lot of algebraic topology is necessary to proof the relations. *Michael Atiyah* found an alternative construction that mainly involves the representation theory of the symmetric groups.

Beginning with a short discussion of the problem in question, its history and different approaches, I will then briefly introduce some basic notions concerning vector bundles, topological K-theory and representation theory of groups. We will see the Adams operations and their relations and roughly sketch Atiyah's construction. Eventually, I will show in detail how their properties are used in order to solve the "Hopf invariant 1" problem.

Duration: approx. 75 minutes. My presentation will be as self-contained as possible; a first course in topology and algebra will be sufficient to follow (most of) the talk.