FRIABLE VALUES OF BINARY FORMS

ANTAL BALOG, VALENTIN BLOMER, CÉCILE DARTYGE, AND GÉRALD TENENBAUM

ABSTRACT. Let $F \in \mathbb{Z}[X, Y]$ be an integral binary form of degree $g \ge 2$, and let

 $\Psi_F(x,y) := \operatorname{card}\{1 \leqslant a, b \leqslant x : P^+(F(a,b)) \leqslant y\}$

where as usual $P^+(n)$ denotes the largest prime factor of n. It is proved that $\Psi_F(x,y) \approx x^2$ for $y = x^{g-2+\varepsilon}$ in general, and $y = x^{1/\sqrt{e}+\varepsilon}$ if g = 3. Better results are obtained if F is reducible.

To the memory of our friend and colleague George Greaves

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