Minisymposium 10 - The use of proof theory in mathematics

Shoenfield = Gödel after Krivine

THOMAS STREICHER (TECHNISCHE UNIVERSITÄT DARMSTADT)

In the 1960s J. Shoenfield came up with a functional interpretation $(-)^S$ of Peano arithmetic (PA). Recently, G. Mints raised the question whether one can express $(-)^S$ as $(A^K)^D$ where D is Gödel's Dialectica interpretation and $(-)^K$ is an appropriately chosen negative translation.
We present such a translation $(-)^K$ going back to J.-L. Krivine and elaborated by B. Reus and T. Streicher, and prove that if

$$A^S \equiv \forall u \exists x A_S(u,x) \quad \text{and} \quad (A^K)^D \equiv \exists f \forall u A_D^K(f,u)\,,$$

then $A_D^K(f,u)$ and $A_S(u,f(u))$ are provably equivalent in HA_ω.
The content of this talk is joint work with Ulrich Kohlenbach.