**Prof. Dr. P. Scholze**
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# Algebraic Geometry I

## 7. Exercise sheet

**Exercise 1 (4 Points):**

Let $S$ be a scheme and let $\mathcal{A}$ be a quasi-coherent $\mathcal{O}_S$-algebra. Prove that the functor

$$(f\colon Y \to S) \mapsto \mathrm{Hom}_{\mathcal{O}_S-alg}(\mathcal{A}, f_*(\mathcal{O}_Y))$$

on the category of schemes over $S$ is representable by a scheme over $S$ (denoted $\underline{\mathrm{Spec}}_{\mathcal{O}_S}\mathcal{A}$).

**Exercise 2 (4 Points):**

Let $R$ be a ring. Compute the global sections of $\mathcal{O}_{\mathbb{P}^n_R}(m)$, $m \in \mathbb{Z}$.

**Exercise 3 (4 Points):**

i) Let $S$ be a scheme and let $\mathcal{E}$ be a vector bundle on $S$. Let $s \in \mathcal{E}(S)$ be a global section. Prove that there exists a unique closed subscheme $V(s)$ of $S$ such that a morphism $f\colon Y \to S$ factors over $V(s)$ if and only if $f^*(s) \in f^*\mathcal{E}(Y)$ is zero.

ii) Recall the Segre embedding $f\colon \mathbb{P}^1_R \times_{\mathrm{Spec}(R)} \mathbb{P}^1_R \subseteq \mathbb{P}^3_R$ from exercise sheet 6. Show that $f$ is a closed embedding, isomorphic to $V(s)$ for a suitable $s$ and $\mathcal{E}$.

**Exercise 4 (4 Points):**

Let $R$ be a ring and set $X := \mathbb{P}^n_R$. Let $\mathcal{M}$ be a quasi-coherent $\mathcal{O}_X$-module of finite type. Show that there exists an $m_0 \in \mathbb{Z}$ such that for every $m \geq m_0$ the $\mathcal{O}_X$-module $\mathcal{M} \otimes_{\mathcal{O}_X} \mathcal{O}_X(m)$ is generated by global sections. If $R$ is noetherian, deduce that every closed subscheme of $X$ is of the form described in exercise 3.i).

To be handed in on: Tuesday, 6. December 2016.