Prof. Dr. P. Scholze **SS 2017**
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Algebraic Geometry II

1. Exercise sheet

Exercise 1 (4 bonus points):

Let A be a ring and let M be an A-module. Recall that M is flat over A if the functor $M \otimes_A -$ is exact.

i) Show that M is flat over A if and only if for every maximal ideal $\mathfrak{m} \subseteq A$ the $A_\mathfrak{m}$-module $M_\mathfrak{m}$ is flat.

ii) Assume that A is a Dedekindring. Show that M is flat over A if and only if it is torsionfree, i.e., if $fm = 0$ for $f \in A \setminus \{0\}$ and $m \in M$, then $m = 0$.

Hint: Use i) to reduce to the case of a discrete valuation ring. In that case, write a torsionfree A-module M as a direct limit of free A-modules and use that tensor products commute with direct limits.

Exercise 2 (4 bonus points):

Let k be a field of characteristic 2 and let $X \subseteq \mathbb{P}^5_k$ (with homogeneous coordinates $x_1, x_2, x_3, y_1, y_2, y_3$) be defined by the equations

$$\begin{aligned} x_1^2 + x_2x_3 + y_1^2 + x_1y_1 = 0 \\ x_2^2 + x_1x_3 + y_2^2 + x_2y_2 = 0 \\ x_3^2 + x_1x_2 + y_3^2 + x_3y_3 = 0. \end{aligned}$$

Prove that every k-rational point of X is smooth.

To be handed in on: Monday, 24. April 2017.