Synthetic spectra and the even filtration: Syllabus

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Contents

1 Schedule

The seminar meets every Wednesday at 14:00-16:00 in the Zeichensaal at Wegelstr. 10.

Date		Topic	Speaker
29 Nov	1	Introduction	Lucas
6 Dec	2	Definition of Syn_E	Emma
13 Dec	3	Cofibre of τ	Sil
20 Dec	4	Filtered models	Lucas
10 Jan	5	Modified Adams spectral sequences	Jack
17 Jan	6	Quivers and the Adams spectral sequence	Liz
24 Jan	7	Multiplicative structures on Moore spectra	Nikolai
31 Jan	8	More Moore spectra	TBD

Schedule of talks

2 Syllabus

2.1 Talk 1: Introduction

This talk is an introduction to the world of synthetic spectra, and their motivation in terms of Adams spectral sequences and Miller's approach to them using homological algebra in spectra. Includes the basics on the construction of synthetic spectra and the setup of the τ-Bockstein spectral sequence. Further, give a sketch of the identification of the generic and special fibres of synthetic spectra and how they fit in the deformation picture, as well as its geometric intuition in terms of filtered spectra. Sketch the filtered model.[1]

References: [Pst22], [Ara], [BHS22a], [Mou21].

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[1] The notes for this talk contain more than was covered.

2.2 Talk 2: Definition of $\mathcal{S}yn_E$

Give a detailed description of the construction of synthetic spectra in Piotr's thesis. In particular, show how we obtain a natural t-structure and symmetric monoidal structure using nice properties of spherical sheaves on the excellent site of finite E-projective spectra. Describe certain variations such as νE-local synthetic spectra and even synthetic spectra.
References: Sections 2,3,5 of [Pst22].

2.3 Talk 3: Cofibre of τ

Identify the special and generic fibres of $\mathcal{S}yn_E$ and how they give rise to the identification between the τ-Bockstein spectral sequence and the Adams spectral sequence. Describe some basic manipulations with these, as well as how one can use these to understand the Adams spectral sequence more closely.
References Section 4 of [Pst22], Appendix A of [BHS22b].

2.4 Talk 4: Filtered models

Describe how synthetic spectra form a monadic adjunction with filtered spectra and how one can give an explicit description in certain cases. Relate this to the story of décalage and the geometric picture of deformations of homotopy theories, in particular, introduce the τ-arithmetic recollement.
References: [Mou21], Appendix C of [BHS22a], [Hed], [Ari21].

2.5 Talk 5: Modified Adams spectral sequences

Discuss how the synthetic setup allows one to construct spectral sequences that do not necessarily arise as the Adams spectral sequence of a given spectrum, yet still converge and give us interesting information. Sketch the construction of synthetic lifts of well known spectra (and their connective models) that arise in this fashion.
References: Forthcoming work of Jack with Christian:)

2.6 Talk 6: Quivers and the Adams spectral sequence

Discuss what happens when we drop the flatness assumption on our base homology theory. Describe how quiver representations offer a descripton of the Adams E^2-page and a modified construction of synthetic spectra based on a class of simples. Illustrate with example.
References: [BP23].

2.7 Talk 7: Multiplicative structures on Moore spectra

Discuss an application of synthetic spectra to the proof that Moore spectra admit more multiplicative structure than expected. If time permits, discuss some consequences of this result.
References: [Bur22]

References

[Ara]   Nersés Aramian. *The Adams spectral sequence*. URL: https://ncatlab.org/nlab/files/AramianANSS.pdf (visited on 12/04/2023).

[Ari21]   Stefano Ariotta. *Coherent cochain complexes and Beilinson t-structures, with an appendix by Achim Krause*. arXiv:2109.01017 [math]. Sept. 2021. URL: http://arxiv.org/abs/2109.01017 (visited on 12/04/2023).

[Bur22]   Robert Burklund. *Multiplicative structures on Moore spectra*. arXiv:2203.14787 [math]. June 2022. DOI: 10.48550/arXiv.2203.14787. URL: http://arxiv.org/abs/2203.14787 (visited on 09/19/2022).

[BHS22a]   Robert Burklund, Jeremy Hahn, and Andrew Senger. *Galois reconstruction of Artin-Tate ₘ-motivic spectra*. arXiv:2010.10325 [math]. June 2022. URL: http://arxiv.org/abs/2010.10325 (visited on 09/14/2022).

[BHS22b]   Robert Burklund, Jeremy Hahn, and Andrew Senger. *On the boundaries of highly connected, almost closed manifolds*. arXiv:1910.14116 [math]. May 2022. DOI: 10.48550/arXiv.1910.14116. URL: http://arxiv.org/abs/1910.14116 (visited on 09/19/2022).

[BP23]   Robert Burklund and Piotr Pstrągowski. *Quivers and the Adams spectral sequence*. arXiv:2305.08231 [math]. May 2023. URL: http://arxiv.org/abs/2305.08231 (visited on 05/30/2023).

[Hed]   Alice Petronella Hedenlund. "Multiplicative Tate Spectral Sequences". URL: https://www.mn.uio.no/math/personer/vit/rognes/theses/hedenlund-thesis.pdf (visited on 12/03/2023).

[Mou21]   Tasos Moulinos. "The geometry of filtrations". en. *Bulletin of the London Mathematical Society* 53.5 (Oct. 2021). arXiv:1907.13562 [math]. ISSN: 0024-6093, 1469-2120. DOI: 10.1112/blms.12512. URL: http://arxiv.org/abs/1907.13562 (visited on 09/14/2022).

[Pst22]   Piotr Pstrągowski. *Synthetic spectra and the cellular motivic category*. arXiv:1803.01804 [math]. Feb. 2022. URL: http://arxiv.org/abs/1803.01804 (visited on 09/14/2022).