Conjectural relation of twisted differentials to Pixton's cycle

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Fix a genus $g \ge 0$, a number $n \ge 0$ of markings, an integer $k \ge 0$ and a partition $\tilde{\mu} = (\tilde{m}_1, \dots, \tilde{m}_n)$ of k(2g - 2 + n).



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Note

We can obtain the partition $\tilde{\mu}$ above from a partition $\mu = (m_1, \dots, m_n)$ of k(2g - 2) by $\tilde{\mu} = (m_1 + k, \dots, m_n + k)$

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Fix a genus $g \ge 0$, a number $n \ge 0$ of markings, an integer $k \ge 0$ and a partition $\tilde{\mu} = (\tilde{m}_1, \dots, \tilde{m}_n)$ of k(2g - 2 + n). Then for every degree $d \ge 0$, Pixton constructs a tautological class $P_g^{d,k}(\tilde{\mu}) \in R^d(\overline{M}_{g,n}) \subset A^d(\overline{M}_{g,n})$.

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Summary : Pixton's cycle

The tautological class $P_g^{d,k}(\widetilde{\mu})$ is defined as an explicit sum over dual graphs Γ and additional combinatorial data of terms of the form

$$\xi_{\Gamma*}\left(\text{polynomial in }\kappa \text{ and }\psi\text{-classes on }\prod_{v\in V(\Gamma)}\overline{M}_{g(v),n(v)}
ight).$$

For $k \ge 1$ and μ not of the form $\mu = k\mu'$ for a nonnegative partition μ' of 2g - 2, we have

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For $k \ge 1$ and μ not of the form $\mu = k\mu'$ for a nonnegative partition μ' of 2g - 2, we have

$$\sum_{\Gamma \text{ star graph}} \sum_{I} \frac{1}{|\operatorname{Aut}(\Gamma)|} C_{\Gamma,I} =$$

where

$$C_{\Gamma,I} = (\xi_{\Gamma})_* \left[\left[\overline{\mathcal{H}}_{g(v_0)}^k (\mu[v_0], -I[v_0] - k) \right] \cdot \prod_{v \in V_{\text{out}}(\Gamma)} \left[\overline{\mathcal{H}}_{g(v)}^1 \left(\frac{\mu[v]}{k}, \frac{I[v] - k}{k} \right) \right] \right]$$

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$$\sum_{\Gamma \text{ star graph}} \sum_{I} \frac{\prod_{e \in E(\Gamma)} I(e)}{k^{|V_{\text{out}}(\Gamma)|}} \frac{1}{|\text{Aut}(\Gamma)|} C_{\Gamma,I} = 2^{-g} P_g^{g,k}(\widetilde{\mu})$$

where

$$C_{\Gamma,I} = (\xi_{\Gamma})_* \left[\left[\overline{\mathcal{H}}_{g(v_0)}^k (\mu[v_0], -I[v_0] - k) \right] \cdot \prod_{v \in V_{\text{out}}(\Gamma)} \left[\overline{\mathcal{H}}_{g(v)}^1 \left(\frac{\mu[v]}{k}, \frac{I[v] - k}{k} \right) \right] \right]$$

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Application: Recursion for $[\overline{\mathcal{H}}_{g}^{k}(\mu)]$

Theorem (FP-Appendix, S.)

Conjecture A effectively determines the classes $[\overline{\mathcal{H}}_{g}^{k}(\mu)]$ for

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Theorem (FP-Appendix, S.)

Conjecture A effectively determines the classes $[\overline{\mathcal{H}}_{a}^{k}(\mu)]$ for

$$\int k \ge 1$$
 and $\mu \ne k \mu'$ with $\mu' \ge 0$ (codim g)

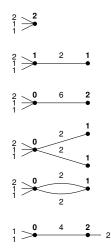
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Theorem (FP-Appendix, S.)

Conjecture A effectively determines the classes $[\overline{\mathcal{H}}_{q}^{k}(\mu)]$ for

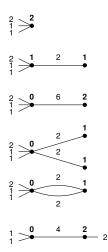
$$egin{cases} k\geq 1 ext{ and } \mu
eq k\mu' ext{ with } \mu'\geq 0 ext{ (codim } g) \ k=1 ext{ and } \mu\geq 0 ext{ (codim } g-1) \end{cases}$$

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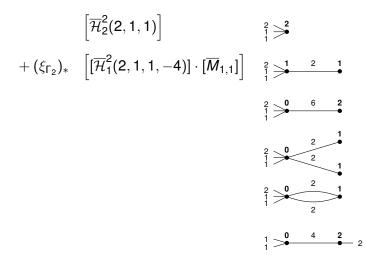
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$$\left[\overline{\mathcal{H}}_{2}^{2}(2,1,1)\right]$$



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$$\begin{bmatrix} \overline{\mathcal{H}}_{2}^{2}(2,1,1) \end{bmatrix} \qquad \stackrel{2}{\underset{1}{2}} \stackrel{2}{\underset{1}{2}} \stackrel{2}{\underset{1}{2}} \stackrel{2}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{2}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{2}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{2}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}} \stackrel{1}{\underset{1}{2}}$$

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$$\begin{bmatrix} \overline{\mathcal{H}}_{2}^{2}(2,1,1) \end{bmatrix} \qquad \stackrel{2}{\underset{1}{2}} \xrightarrow{2}{\underset{1}{2}} \xrightarrow{2}{\underset{1}{2}} \\ + (\xi_{\Gamma_{2}})_{*} \begin{bmatrix} [\overline{\mathcal{H}}_{1}^{2}(2,1,1,-4)] \cdot [\overline{\mathcal{M}}_{1,1}] \end{bmatrix} \qquad \stackrel{2}{\underset{1}{2}} \xrightarrow{1}{\underset{1}{2}} \xrightarrow{1}{\underset{1}{2$$

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Take Conjecture A for $\mu^+ = (3, -1)$ on $\overline{M}_{2,2}$

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Take Conjecture A for $\mu^+ = (3, -1)$ on $\overline{M}_{2,2}$

$$\begin{bmatrix} \overline{\mathcal{H}}_{2}^{1}(3,-1) \end{bmatrix} \xrightarrow{3}{}_{-1}^{3} \xrightarrow{2} \\ + (\xi_{\Gamma_{2}})_{*} \begin{bmatrix} [\overline{\mathcal{H}}_{1}^{1}(3,-1,-2)] \cdot [\overline{\mathcal{M}}_{1,1}] \end{bmatrix} \xrightarrow{3}{}_{-1}^{1} \xrightarrow{1}{}_{-1}^{1} \xrightarrow{1}{}_{-1}^{1} \xrightarrow{1}{}_{-1}^{1} \\ + \frac{1}{2} (\xi_{\Gamma_{3}})_{*} \begin{bmatrix} [\overline{\mathcal{M}}_{0,4}] \cdot [\overline{\mathcal{M}}_{1,1}] \cdot [\overline{\mathcal{M}}_{1,1}] \end{bmatrix} \xrightarrow{3}{}_{-1}^{0} \xrightarrow{1}{}_{-1}^{1} \xrightarrow{0}{}_{-1}^{1} \xrightarrow{1}{}_{-1}^{1} \\ + \frac{1}{2} (\xi_{\Gamma_{4}})_{*} \begin{bmatrix} [\overline{\mathcal{M}}_{0,4}] \cdot [\overline{\mathcal{M}}_{1,2}] \end{bmatrix} \xrightarrow{3}{}_{-1}^{0} \xrightarrow{1}{}_{-1}^{1} \xrightarrow{1}{}_{-1}^{1} \xrightarrow{1}{}_{-1}^{1} \\ + 3 (\xi_{\Gamma_{5}})_{*} \begin{bmatrix} [\overline{\mathcal{M}}_{0,3}] \cdot [\overline{\mathcal{H}}_{2}^{1}(2)] \end{bmatrix} \xrightarrow{3}{}_{-1}^{0} \xrightarrow{3}{}_{-1}^{2} \xrightarrow{0}{}_{-1}^{3} \xrightarrow{0}{}_{-1}^{2} \xrightarrow{1}{}_{-1}^{1} \\ = \frac{1}{4} P_{2}^{2,1}(4,0)$$

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Take Conjecture A for $\mu^+ = (3, -1)$ on $\overline{M}_{2,2}$ and push forward under the forgetful map $\epsilon : \overline{M}_{2,2} \to \overline{M}_{2,1}$ of the second point

$$\begin{bmatrix} \overline{\mathcal{H}}_{2}^{1}(3,-1) \end{bmatrix} \xrightarrow{3}{-1} \geq^{2}$$

$$+ (\xi_{\Gamma_{2}})_{*} \begin{bmatrix} [\overline{\mathcal{H}}_{1}^{1}(3,-1,-2)] \cdot [\overline{\mathcal{M}}_{1,1}] \end{bmatrix} \xrightarrow{3}{-1} \stackrel{1}{\longrightarrow} \stackrel{1}{$$

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$$\epsilon_{*} \quad \left[\overline{\mathcal{H}}_{2}^{1}(3,-1)\right] \qquad \begin{array}{c} \overset{3}{_{-1}} \gg^{2} \\ + \epsilon_{*}(\xi_{\Gamma_{2}})_{*} \quad \left[[\overline{\mathcal{H}}_{1}^{1}(3,-1,-2)] \cdot [\overline{M}_{1,1}]\right] \qquad \begin{array}{c} \overset{3}{_{-1}} \gg^{1} \xrightarrow{1} & \overset{1}{_{-1}} \xrightarrow{1} \\ + \frac{1}{2}\epsilon_{*}(\xi_{\Gamma_{3}})_{*} \quad \left[[\overline{M}_{0,4}] \cdot [\overline{M}_{1,1}] \cdot [\overline{M}_{1,1}]\right] \qquad \begin{array}{c} \overset{3}{_{-1}} \gg^{0} \xrightarrow{1} \xrightarrow{1} & \overset{1}{_{-1}} \xrightarrow{1} \\ + \frac{1}{2}\epsilon_{*}(\xi_{\Gamma_{4}})_{*} \quad \left[[\overline{M}_{0,4}] \cdot [\overline{M}_{1,2}]\right] \qquad \begin{array}{c} \overset{3}{_{-1}} \gg^{0} \xrightarrow{1} \xrightarrow{1} \\ & \overset{3}{_{-1}} \gg^{0} \xrightarrow{1} \xrightarrow{1} \end{array} \qquad \begin{array}{c} \overset{3}{_{-1}} \xrightarrow{0} \xrightarrow{1} \xrightarrow{1} \xrightarrow{1} \\ & \overset{3}{_{-1}} \gg^{0} \xrightarrow{3} \xrightarrow{2} \\ & \overset{3}{_{-1}} \gg^{0} \xrightarrow{3} \xrightarrow{2} \end{array} \qquad \\ = & \frac{1}{4}\epsilon_{*}P_{2}^{2,1}(4,0) \end{array}$$

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$$\epsilon_{*}(\xi_{\Gamma_{2}})_{*} \begin{bmatrix} [\overline{\mathcal{H}}_{1}^{1}(3,-1,-2)] \cdot [\overline{\mathcal{M}}_{1,1}] \end{bmatrix} \xrightarrow{3}{-1} \xrightarrow{1}{-1} \xrightarrow{1} \xrightarrow{1}{-1} \xrightarrow{1}{-1} \xrightarrow{1} \xrightarrow{$$

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 $\frac{3}{1} \rightarrow \frac{2}{4}$

Take Conjecture A for $\mu^+ = (3, -1)$ on $\overline{M}_{2,2}$ and push forward under the forgetful map $\epsilon : \overline{M}_{2,2} \to \overline{M}_{2,1}$ of the second point

 $\epsilon_*(\xi_{\Gamma_2})_* \quad \left| [\overline{\mathcal{H}}_1^1(3, -1, -2)] \cdot [\overline{M}_{1,1}] \right| \xrightarrow{3} \underbrace{1}_{} \underbrace{1}_{}$ $+\frac{1}{2}\epsilon_*(\xi_{\Gamma_2})_*$ $[\overline{M}_{04}] \cdot [\overline{M}_{11}] \cdot [\overline{M}_{11}]$ _3 _0 $+\frac{1}{2}\epsilon_*(\xi_{\Gamma_4})_*$ $[[\overline{M}_{0,4}] \cdot [\overline{M}_{1,2}]]$ $\left|\overline{\mathcal{H}}_{2}^{1}(2)\right|$ + 3 3 0 3 2 $= \frac{1}{4}\epsilon_* P_2^{2,1}(4,0)$

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 $\frac{3}{1} \rightarrow \frac{2}{4}$



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• g = 0 trivial (1 = 1)



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- *g* = 0 trivial (1 = 1)
- g = 1 (FP-Appendix)

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- g = 0 trivial (1 = 1)
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• k = 1 and $\mu = (3, -1), (2, 1, -1)$ (FP-Appendix)

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- g = 0 trivial (1 = 1)
- g = 1 (FP-Appendix)
- *g* = 2
 - k = 1 and μ = (3, −1), (2, 1, −1) (FP-Appendix)
 Note: H¹₂(μ) = Ø in both cases

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- g = 0 trivial (1 = 1)
- g = 1 (FP-Appendix)
- *g* = 2
 - k = 1 and $\mu = (3, -1), (2, 1, -1)$ (FP-Appendix) Note: $\mathcal{H}_2^1(\mu) = \emptyset$ in both cases
 - k = 2 and $\mu = (3, 1), (2, 1, 1)$ (S)

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•
$$g = 0$$
 trivial $(1 = 1)$
• $g = 1$ (FP-Appendix)
• $g = 2$
• $k = 1$ and $\mu = (3, -1), (2, 1, -1)$ (FP-Appendix)
Note: $\mathcal{H}_2^1(\mu) = \emptyset$ in both cases
• $k = 2$ and $\mu = (3, 1), (2, 1, 1)$ (S)
• $\mathcal{H}_2^2(3, 1) = \emptyset$
• $\mathcal{H}_2^2(2, 1, 1) = \begin{cases} q \text{ Weierstrass point,} \\ (C, q, p_1, p_2) : p_1, p_2 \text{ hyperelliptic con-} \\ jugate \end{cases}$

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$$\begin{bmatrix} \overline{\mathcal{H}}_{2}^{2}(2,1,1) \end{bmatrix} \xrightarrow{2}{1} \xrightarrow{2}{\bullet} + (\xi_{\Gamma_{2}})_{*} \begin{bmatrix} [\overline{\mathcal{H}}_{1}^{2}(2,1,1,-4)] \cdot [\overline{M}_{1,1}] \end{bmatrix} \xrightarrow{2}{1} \xrightarrow{1}{\bullet} + 3(\xi_{\Gamma_{3}})_{*} \begin{bmatrix} [\overline{M}_{0,4}] \cdot [\overline{\mathcal{H}}_{2}^{1}(2)] \end{bmatrix} \xrightarrow{2}{1} \xrightarrow{0} \xrightarrow{6}{\bullet} \xrightarrow{2}{\bullet} + \frac{1}{2}(\xi_{\Gamma_{4}})_{*} \begin{bmatrix} [\overline{M}_{0,5}] \cdot [\overline{M}_{1,1}] \cdot [\overline{M}_{1,1}] \end{bmatrix} \xrightarrow{2}{1} \xrightarrow{0} \xrightarrow{2}{\bullet} \xrightarrow{1}{\bullet} + (\xi_{\Gamma_{5}})_{*} \begin{bmatrix} [\overline{M}_{0,5}] \cdot [\overline{M}_{1,2}] \end{bmatrix} \xrightarrow{2}{1} \xrightarrow{0} \xrightarrow{2}{\bullet} \xrightarrow{1}{\bullet} + 2(\xi_{\Gamma_{6}})_{*} \begin{bmatrix} [\overline{M}_{0,3}] \cdot [\overline{\mathcal{H}}_{2}^{1}(1,1)] \end{bmatrix} \xrightarrow{1}{1} \xrightarrow{0} \xrightarrow{4}{\bullet} \xrightarrow{2}{\bullet} 2 = \frac{1}{4}P_{2}^{2,2}(4,3,3)$$

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What if
$$\mu = k\mu'$$
 with $\mu' \ge 0$?
 $\left[\overline{\mathcal{H}}_{g}^{k}(\mu)\right] = \left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right] + \left[\overline{\mathcal{H}}_{g}^{k}(\mu)'\right]$

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 $\left[\overline{\mathcal{H}}_{g}^{k}(\mu)\right] = \left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right]^{\text{vir}} + \left[\overline{\mathcal{H}}_{g}^{k}(\mu)'\right] \in A^{g}(\overline{M}_{g,n})$

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(1) Formula from Conjecture A for μ, k :
 $\left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right]^{\operatorname{vir}} + \left[\overline{\mathcal{H}}_{g}^{k}(\mu)'\right] + \sum_{(\Gamma, I) \text{ nontrivial}} \operatorname{Cont}_{g,\mu}^{k}(\Gamma, I) = 2^{-g} P_{g}^{g,k}(\widetilde{\mu})$

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(1) Formula from Conjecture A for μ, k :
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(2) Formula from Conjecture A for $\frac{\mu}{k}, 1$:
 $\left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right]^{\operatorname{vir}} + \sum_{(\Gamma', l') \text{ nontrivial}} \operatorname{Cont}_{g,\mu/k}^{1}(\Gamma', l') = 2^{-g}P_{g}^{g,1}(\widetilde{\mu/k})$

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Conjecture A'

What if
$$\mu = k\mu'$$
 with $\mu' \ge 0$?
 $\left[\overline{\mathcal{H}}_{g}^{k}(\mu)\right] = \left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right]^{\operatorname{vir}} + \left[\overline{\mathcal{H}}_{g}^{k}(\mu)'\right] \in A^{g}(\overline{M}_{g,n})$
(1) Formula from Conjecture A for μ, k :
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(2) Formula from Conjecture A for $\frac{\mu}{k}, 1$:
 $\left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right]^{\operatorname{vir}} + \sum_{(\Gamma',l') \text{ nontrivial}} \operatorname{Cont}_{g,\mu/k}^{1}(\Gamma', l') = 2^{-g}P_{g}^{g,1}(\widetilde{\mu/k})$
Idea: Take (2) as a definition of $\left[\overline{\mathcal{H}}_{g}^{1}(\frac{\mu}{k})\right]^{\operatorname{vir}}$
 $\Longrightarrow \operatorname{Conjecture} A' = (1) \text{ for } k > 1, \mu = k\mu' \text{ with } \mu' \ge 0$

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Evidence for Conjecture A'

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• g = 0 no nontrivial examples possible

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- g = 0 no nontrivial examples possible
- g = 1 implies $\mu = (0, ..., 0)$, check on formulas



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- g = 0 no nontrivial examples possible
- g = 1 implies $\mu = (0, ..., 0)$, check on formulas
- g = 2, k = 2 and $\mu = (4), (2, 2)$

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•
$$g = 2, k = 2$$
 and $\mu = (4), (2, 2)$

•
$$\mathcal{H}_{2}^{2}(4)' = \emptyset$$

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$$g = 2, k = 2$$
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•
$$\mathcal{H}_2^2(4)' = \emptyset$$

 \$\mathcal{H}_2^2(2,2)' = {(C, p, q) : p, q Weierstrass points}\$ Class computed by Tarasca in 2015

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- g = 0 no nontrivial examples possible
- g = 1 implies $\mu = (0, ..., 0)$, check on formulas

•
$$g = 2, k = 2$$
 and $\mu = (4), (2, 2)$

Again, Conjecture A' gives an effective way to compute the classes $\left[\overline{\mathcal{H}}_g^k(\mu)'\right]$.

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Thank you for your attention.

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Let Γ be a stable graph of genus *g* with *n* legs. A *k*-weighting mod *r* of Γ is a function on the set of half-edges

$$w: H(\Gamma) \rightarrow \{0, 1, \ldots, r-1\}$$

satisfying

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satisfying

i) for the leg h_i corresponding to marking i

 $w(h_i) = \widetilde{m}_i \mod r$,

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i) for the leg h_i corresponding to marking i

 $w(h_i) = \widetilde{m}_i \mod r$,

ii) for all half-edges h, h' forming an edge $w(h) + w(h') = 0 \mod r$

Let Γ be a stable graph of genus *g* with *n* legs. A *k*-weighting mod *r* of Γ is a function on the set of half-edges

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i) for the leg h_i corresponding to marking i

$$w(h_i) = \widetilde{m}_i \mod r$$
,

ii) for all half-edges h, h' forming an edge $w(h) + w(h') = 0 \mod r$

ii) for all vertices v of Г

$$\sum_{v(h)=v} w(h) = k(2g(v) - 2 + n(v)) \mod r$$

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For each positive integer *r*, let $P_g^{d,r,k}(\tilde{\mu})$ be the degree *d* component of the tautological class

$$\sum_{\Gamma,w} \frac{1}{|\operatorname{Aut}(\Gamma)|} \frac{1}{r^{h^{1}(\Gamma)}} \xi_{\Gamma*} \Big[\prod_{v \in V(\Gamma)} e^{-k^{2} \kappa_{1}(v)} \prod_{i=1}^{n} e^{\widetilde{m}_{i}^{2} \psi_{h_{i}}} \\ \prod_{e=(h,h') \in E(\Gamma)} \frac{1 - e^{-w(h)w(h')(\psi_{h} + \psi_{h'})}}{\psi_{h} + \psi_{h'}} \Big]$$

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$$\sum_{\Gamma,w} \frac{1}{|\operatorname{Aut}(\Gamma)|} \frac{1}{r^{h^{1}(\Gamma)}} \xi_{\Gamma*} \Big[\prod_{v \in V(\Gamma)} e^{-k^{2} \kappa_{1}(v)} \prod_{i=1}^{n} e^{\widetilde{m}_{i}^{2} \psi_{h_{i}}} \\ \prod_{e=(h,h') \in E(\Gamma)} \frac{1 - e^{-w(h)w(h')(\psi_{h} + \psi_{h'})}}{\psi_{h} + \psi_{h'}} \Big]$$

Proposition/Definition (Pixton)

 $P_g^{d,r,k}(\widetilde{\mu}) \in R^d(\overline{M}_{g,n})$ is polynomial in r for $r \gg 0$. Let $P_g^{d,k}(\widetilde{\mu}) \in R^d(\overline{M}_{g,n})$ be the value of this polynomial at r = 0.

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